Attendance Problems. Solve for x.

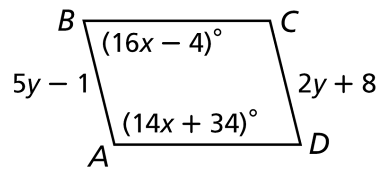
1. $16x - 3 = 12x + 13$

2. $2x - 4 = 90$

***ABCD* is a parallelogram. Find each measure.**

3. CD

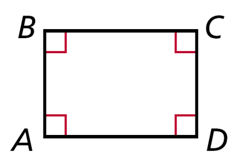
4. $m\measuredangle C$



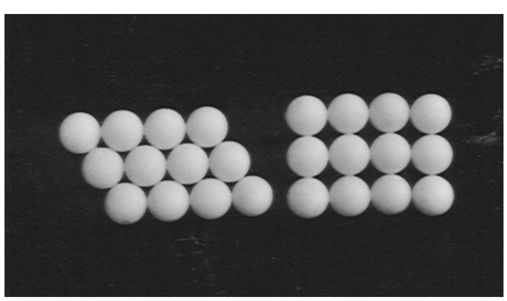
- I can prove and apply properties of rectangles, rhombuses, and squares.
- I can use properties of rectangles, rhombuses, and squares to solve problems.

Common Core: CC.9-12.G.CO.11 Prove theorems about parallelograms.

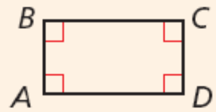
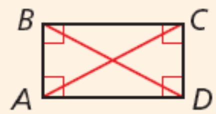
A second type of special quadrilateral is a *rectangle*. A **rectangle** is a quadrilateral with four right angles.



Rectangle *ABCD*

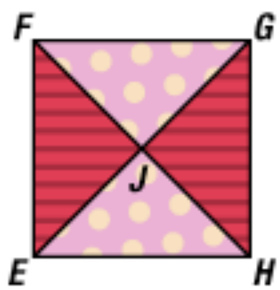


Theorems — Properties of Rectangles

	THEOREM	HYPOTHESIS	CONCLUSION
6-4-1	If a quadrilateral is a rectangle, then it is a parallelogram. (rect. → ▱)		*ABCD* is a parallelogram.
6-4-2	If a parallelogram is a rectangle, then its diagonals are congruent. (rect. → diags. ≅)		$\overline{AC} \cong \overline{BD}$

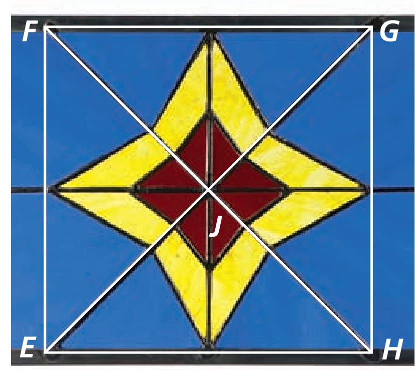
<u>Refer to video example 1.</u>

In this rectangular patch, the quilter cut the strips so that *EH* = 12 in. and *EG* = 20 in. Find *FG* and *FJ*.



1 *Craft Application*

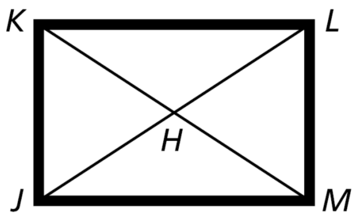
An artist connects stained glass pieces with lead strips. In this rectangular window, the strips are cut so that $FG = 24$ in. and $FH = 34$ in. Find JG.



$\overline{EG} \cong \overline{FH}$	*Rect. → diags. ≅*
$EG = FH = 34$	*Def. of ≅ segs.*
$JG = \frac{1}{2}EG$	*▱ → diags. bisect each other*
$JG = \frac{1}{2}(34) = 17$ in.	*Substitute and simplify.*

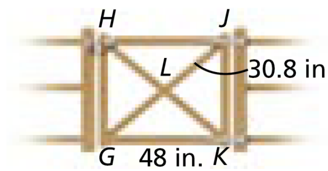
Q: What did the rhombus say to the square?
A: Lean on me, friend!

Example 1. A woodworker constructs a rectangular picture frame so that $JK = 50$ cm and $JL = 86$ cm. Find HM.

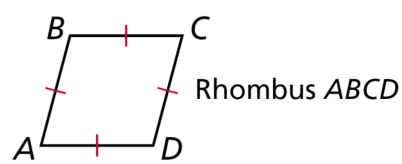


Guided Practice. The rectangular gate has diagonal braces. Find each length.

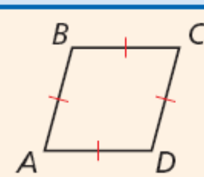
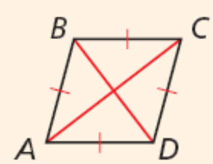
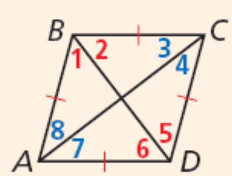
5. HJ **6.** HK



A *rhombus* is another special quadrilateral. A **rhombus** is a quadrilateral with four congruent sides.



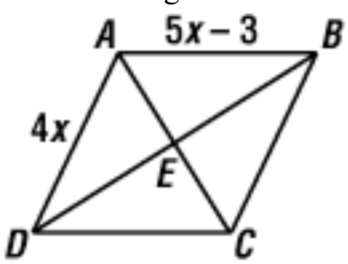
Theorems: Properties of Rhombuses

THEOREM		HYPOTHESIS	CONCLUSION
6-4-3	If a quadrilateral is a rhombus, then it is a parallelogram. (rhombus → ▱)		*ABCD* is a parallelogram.
6-4-4	If a parallelogram is a rhombus, then its diagonals are perpendicular. (rhombus → diags. ⊥)		$\overline{AC} \perp \overline{BD}$
6-4-5	If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. (rhombus → each diag. bisects opp. ∠s)		$\angle 1 \cong \angle 2$ $\angle 3 \cong \angle 4$ $\angle 5 \cong \angle 6$ $\angle 7 \cong \angle 8$

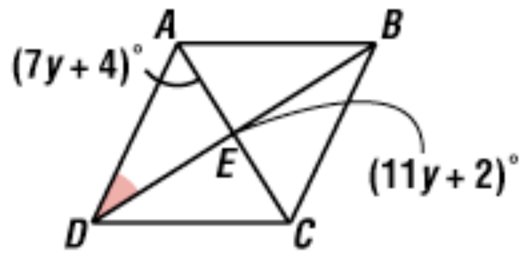
"What is wanted is not the will to believe, but the will to find out, which is the exact opposite." -- *Bertrand Russell*, Sceptical_Essays, 1928

<u>Refer to video example 2.</u>

A. Find the length of CD.

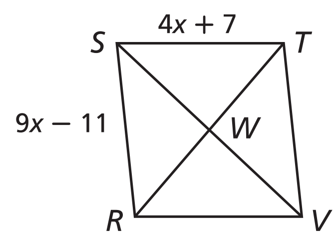


B. Find $m\measuredangle ADB$



2 Using Properties of Rhombuses to Find Measures

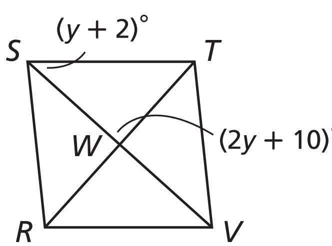
***RSTV* is a rhombus. Find each measure.**



A VT

$ST = SR$	*Def. of rhombus*
$4x + 7 = 9x - 11$	*Substitute the given values.*
$18 = 5x$	*Subtract 4x from both sides and add 11 to both sides.*
$3.6 = x$	*Divide both sides by 5.*
$VT = ST$	*Def. of rhombus*
$VT = 4x + 7$	*Substitute 4x + 7 for ST.*
$VT = 4(3.6) + 7 = 21.4$	*Substitute 3.6 for x and simplify.*

***RSTV* is a rhombus. Find each measure.**

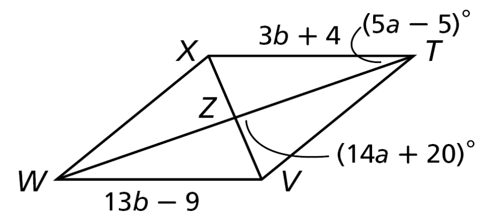


B $m\angle WSR$

$m\angle SWT = 90°$	*Rhombus → diags. ⊥*
$2y + 10 = 90$	*Substitute 2y + 10 for m∠SWT.*
$y = 40$	*Subtract 10 from both sides and divide both sides by 2.*
$m\angle WSR = m\angle TSW$	*Rhombus → each diag. bisects opp. ∠s*
$m\angle WSR = (y + 2)°$	*Substitute y + 2 for m∠TSW.*
$m\angle WSR = (40 + 2)° = 42°$	*Substitute 40 for y and simplify.*

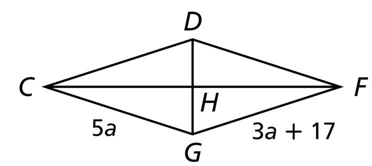
<u>Example 2</u>. TVWX is a rhombus. Find each measure.

A. TV **B.** $m\measuredangle VTZ$



<u>Guided Practice</u>. CDFG is a rhombus.

7. Find the length of CD.

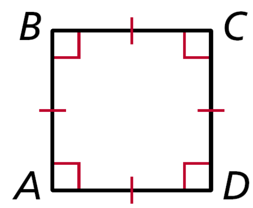


8. Find $m\measuredangle GCH$ if $m\measuredangle GCD = (b+3)°$ & $m\measuredangle CDF = (6b-40)°$

A **<u>square</u>** is a quadrilateral with four right angles and four congruent sides. In the exercises, you will show that a square is a parallelogram, a rectangle, and a rhombus. So a square has the properties of all three.

Helpful Hint

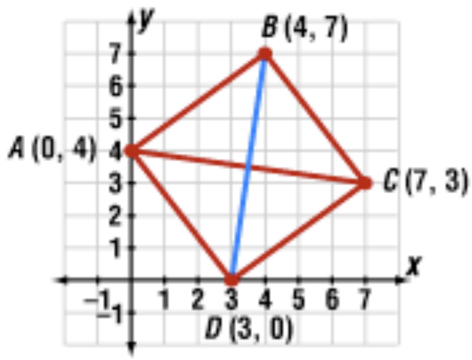
Rectangles, rhombuses, and squares are sometimes referred to as *special parallelograms*.



Square *ABCD*

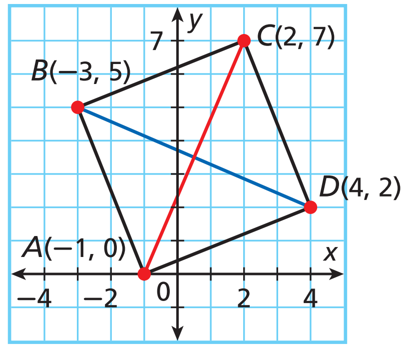
Refer to video example 3.

Show that the diagonals of square ABCD are congruent perpendicular bisectors of each other.



3 Verifying Properties of Squares

Show that the diagonals of square *ABCD* are congruent perpendicular bisectors of each other.



Step 1 Show that $\overline{AC}$ and $\overline{BD}$ are congruent.

$$AC = \sqrt{[2-(-1)]^2 + (7-0)^2} = \sqrt{58}$$

$$BD = \sqrt{[4-(-3)]^2 + (2-5)^2} = \sqrt{58}$$

Since $AC = BD$, $\overline{AC} \cong \overline{BD}$.

Step 2 Show that $\overline{AC}$ and $\overline{BD}$ are perpendicular.

$$\text{slope of } \overline{AC} = \frac{7-0}{2-(-1)} = \frac{7}{3}$$

$$\text{slope of } \overline{BD} = \frac{2-5}{4-(-3)} = \frac{-3}{7} = -\frac{3}{7}$$

Since $\left(\frac{7}{3}\right)\left(-\frac{3}{7}\right) = -1$, $\overline{AC} \perp \overline{BD}$.

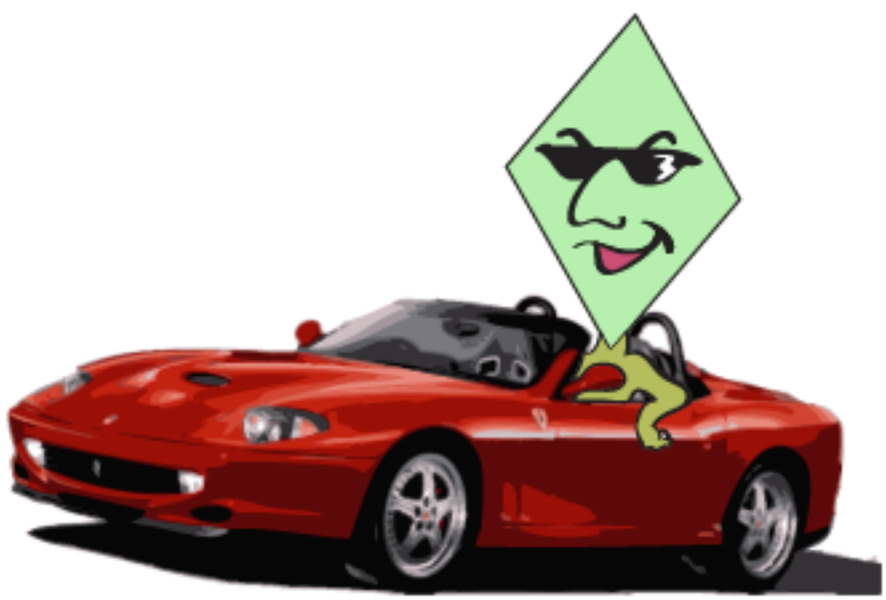
Step 3 Show that $\overline{AC}$ and $\overline{BD}$ bisect each other.

$$\text{mdpt. of } \overline{AC}: \left(\frac{-1+2}{2}, \frac{0+7}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$$

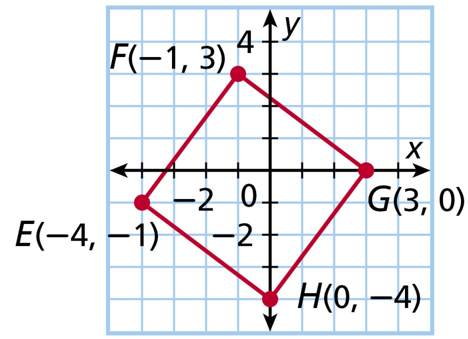
$$\text{mdpt. of } \overline{BD}: \left(\frac{-3+4}{2}, \frac{5+2}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$$

Since $\overline{AC}$ and $\overline{BD}$ have the same midpoint, they bisect each other.

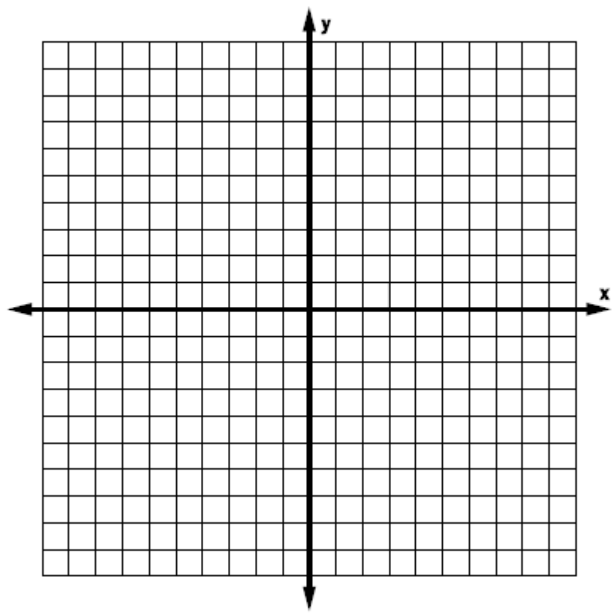
The diagonals are congruent perpendicular bisectors of each other.



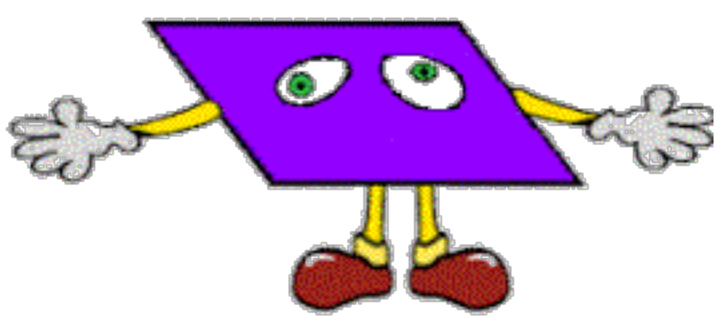
<u>Example 3</u>. Show that the diagonals of square *EFGH* are congruent perpendicular bisectors of each other.



9. <u>Guided Practice</u>. The vertices of square *STVW* are *S*(–5, –4), *T*(0, 2), *V*(6, –3) , and *W*(1, –9) . Show that the diagonals of square *STVW* are congruent perpendicular bisectors of each other.



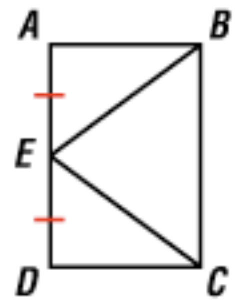
6-4 Properties of Special Parallelograms *(p 424)* 11, 13-16.



<u>Refer to video example 4.</u>

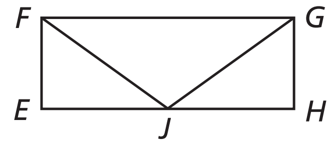
Given:
ABCD is a rectangle.
E is the midpoint of $\overline{AD}$.

Prove:
$\triangle BCE$ is isosceles.



4 Using Properties of Special Parallelograms in Proofs

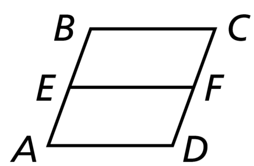
Given: $EFGH$ is a rectangle. J is the midpoint of $\overline{EH}$.
Prove: $\triangle FJG$ is isosceles.



Proof:

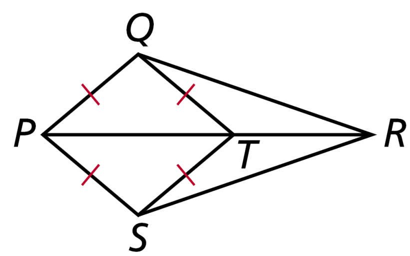
Statements	Reasons
1. $EFGH$ is a rectangle. J is the midpoint of $\overline{EH}$.	**1.** Given
2. $\angle E$ and $\angle H$ are right angles.	**2.** Def. of rect.
3. $\angle E \cong \angle H$	**3.** Rt. $\angle \cong$ Thm.
4. $EFGH$ is a parallelogram.	**4.** Rect. $\rightarrow \square$
5. $\overline{EF} \cong \overline{HG}$	**5.** $\square \rightarrow$ opp. sides $\cong$
6. $\overline{EJ} \cong \overline{HJ}$	**6.** Def. of mdpt.
7. $\triangle FJE \cong \triangle GJH$	**7.** SAS *Steps 3, 5, 6*
8. $\overline{FJ} \cong \overline{GJ}$	**8.** CPCTC
9. $\triangle FJG$ is isosceles.	**9.** Def. of isosc. $\triangle$

<u>Example 4</u>.
Given: $ABCD$ is a rhombus. E is the midpoint of $\overline{AB}$, and F is the midpoint of $\overline{CD}$.
Prove: AEFD is a parallelogram.



<u>Guided Practice</u>.

Given: PQTS is a rhombus with diagonal $\overline{PR}$.
Prove: $\overline{RQ} \cong \overline{RS}$



6-4 Properties of Special Parallelograms *(p 424)* 11, 13-17, 34, 36, 43, 44.

parallelogram

a quadrilateral with opposite sides that are parallel and congruent